Rational (Fractional) Exponents

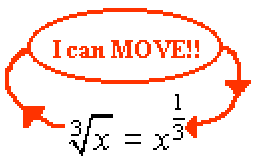
Rational (fractional) exponents are an alternate way to express roots!

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} \;\; ; \; (n \neq 0)$$

We're talking radicals here!

Notice: The denominator of the rational exponent becomes the index of the radical, and the numerator becomes the exponent of the radicand (expression inside the radical).

When you are dealing with a radical expression, you can convert it to an expression containing a rational (fractional) power. This conversion may make the problem easier to solve.



Specifically, $\sqrt[3]{x} = x^{\frac{1}{3}}$, or in general:

$$\sqrt[n]{x^m} = x^{\frac{m}{n}} = \left(\sqrt[n]{x}\right)^m$$

Recall how to simplify radicals:

$\sqrt{36x^2y^4} = 6xy^2$ or $\sqrt[3]{-27a^3b^6} = -3ab^2$

Let's look at these two problems in a new light! When asked to simplify these radicals, it is often **easier to rewrite the radicals** using rational exponents and solve the problems by dealing with the laws of exponents.

Notice how applying the rules for dealing with the exponents makes quick work of the variables.

$$\sqrt{36x^2y^4} = \left(36x^2y^4\right)^{\frac{1}{2}} = 36^{\frac{1}{2}} \bullet (x^2)^{\frac{1}{2}} \bullet (y^4)^{\frac{1}{2}} = 6xy^2$$

$$\sqrt[3]{-27a^3b^6} = \left(-27a^3b^6\right)^{\frac{1}{3}} = -27^{\frac{1}{3}} \bullet (a^3)^{\frac{1}{3}} \bullet (b^6)^{\frac{1}{3}} = -3ab^2$$

Look at these examples:

(1) $\sqrt{x^3} = x^{\frac{3}{2}}$	(2) $\sqrt[3]{a^9} = a^{\frac{9}{3}} = a^3$	(3) $\sqrt[5]{2^{10}} = 2^{\frac{10}{5}} = 2^2 = 4$

When dealing with rational exponents, the Rules for Exponents are still valid!!!

Rule	Example
$x^a \bullet x^b = x^{a+b}$	$\sqrt[3]{x^5} \bullet \sqrt[3]{x^4} = x^{\frac{5}{3}} \bullet x^{\frac{4}{3}} = x^{\frac{5}{3}+\frac{4}{3}} = x^{\frac{9}{3}} = x^3$
$\dfrac{x^a}{x^b} = x^{a-b}$	$\dfrac{\sqrt{a^3}}{\sqrt[4]{a^5}} = \dfrac{a^{\frac{3}{2}}}{a^{\frac{5}{4}}} = a^{\frac{6}{4}-\frac{5}{4}} = a^{\frac{1}{4}} = \sqrt[4]{a}$
$\left(x^a\right)^b = x^{ab}$	$\left(\sqrt{x}\right)^4 = \left(x^{\frac{1}{2}}\right)^4 = x^{\frac{1}{2}\bullet\frac{4}{1}} = x^{\frac{4}{2}} = x^2$

Rationalizing radical denominators may often be accomplished more easily by using rational exponents. Look at this example, solved two ways.	**Simplify:** $\dfrac{3}{\sqrt[3]{3}}$
Solved by Rationalizing the Denominator $\dfrac{3}{\sqrt[3]{3}} = \dfrac{3 \cdot \sqrt[3]{3^2}}{\sqrt[3]{3} \cdot \sqrt[3]{3^2}} = \dfrac{3\sqrt[3]{3^2}}{\sqrt[3]{3^3}} = \dfrac{3\sqrt[3]{3^2}}{3} = \sqrt[3]{3^2}$	**Solved by Using Rational Exponents** $\dfrac{3}{\sqrt[3]{3}} = \dfrac{3}{3^{\frac{1}{3}}} = 3^{1-\frac{1}{3}} = 3^{\frac{2}{3}} = \sqrt[3]{3^2}$

Check out how these problems are done using rational exponents:

Evaluate: $\left(\sqrt[4]{16}\right)^2$	$\left(\sqrt[4]{16}\right)^2 = \left(16^{\frac{1}{4}}\right)^2 = 16^{\frac{1}{4}\bullet\frac{2}{1}} = 16^{\frac{2}{4}} = 16^{\frac{1}{2}} = \sqrt{16} = 4$
Evaluate: $8^{\frac{2}{3}} \bullet 8^{-\frac{1}{3}}$	$8^{\frac{2}{3}} \bullet 8^{-\frac{1}{3}} = 8^{\frac{2}{3}-\frac{1}{3}} = 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$